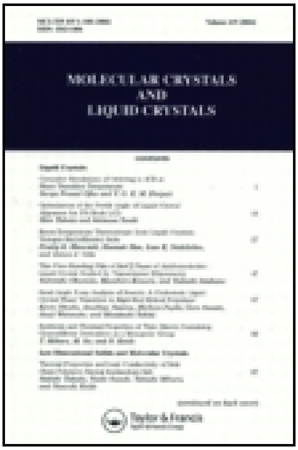

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Response of a Nematic Liquid Crystal to an Acoustic Field

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Response of a Nematic Liquid Crystal to an Acoustic Field

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The type of equations which describe a nematic liquid crystal in an acoustic field are reviewed. Simple examples show that solutions which *allow* for both an acoustic streaming component and an instability can be expected. The response time of the acoustic streaming component is examined via a normal mode analysis.

I INTRODUCTION

The recent literature attests to a growing interest in acoustically induced structures in liquid crystals.[1] The resulting optical effects are of potential use in display devices—for example as acoustic cameras.[2] Theoretical explanations have, until recently, been at odds with a number of experimental observations. Early attempts[3,4] attributed the sudden appearance of the "new" optical character for the system (see Figure 1) to the occurrence of an instability in the steady state regime generated by the imposed acoustic field. Unfortunately, the acoustic power, which these models predicted would bring about an instability, was as much as two orders of magnitude greater than the power which experiment showed would generate the new optical pattern.[5] More recently, experimental results have been explained satisfactorily[6,7] in terms of the gradual build up of an acoustic streaming pattern (see Figure 2). In this explanation the "sudden" appearance of the optical pattern is simply a function of the complicated dependence of the optical effect on the acoustic streaming. This is illustrated in Figure 3. One

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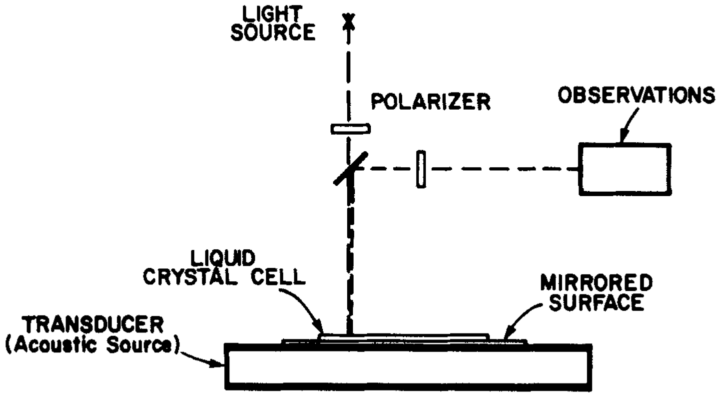

FIGURE 1 Experimental configuration for observation of the acusto-optical effect.

should emphasize that the acoustic streaming pattern invoked in this explanation eventually becomes unstable for, at sufficiently high input power, a turbulent flow pattern obtains with a concomitant sudden change in optical pattern.

A description which accommodates both instabilities and acoustic streaming would appear to be of some interest. To this end we review the mathematical equations which have been employed to describe liquid

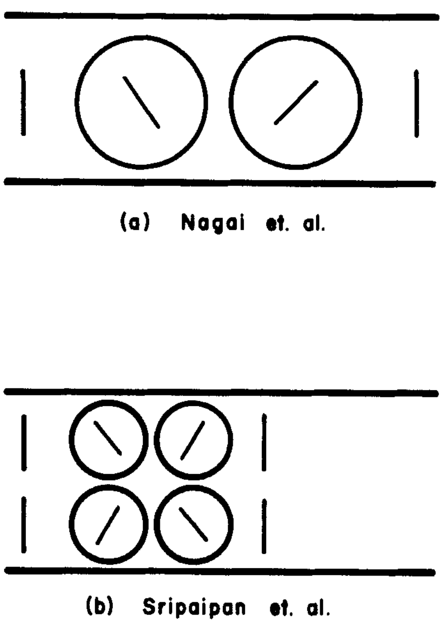

FIGURE 2 Acoustic streaming pattern in a nematic liquid crystal.

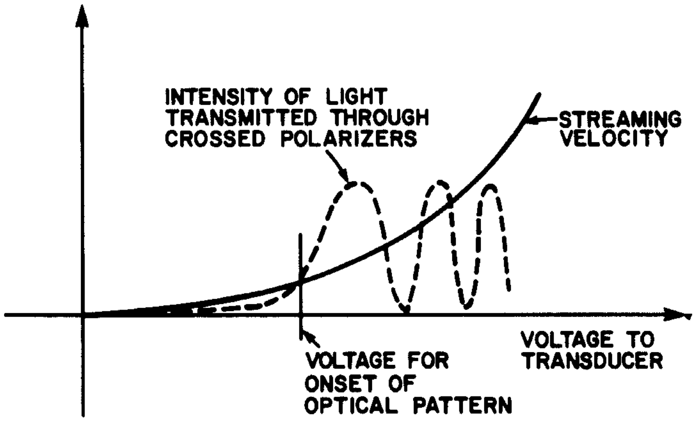

FIGURE 3 Optical and streaming patterns as a function input voltage.

crystals in an acoustic field. With some simple examples we show that, as the sound field is increased, one can obtain solutions which allow for a steady growth of the acoustic streaming component as well as for an instability. [We do not, in this work, examine which phenomena would be observed—this would depend on the experimental probe, the precise boundary conditions, the particular liquid crystal and on the acoustic field strength].

In practical applications of the acousto-optic effect, the response and decay time are of some importance. Their measurement and sensitivity to the acoustic field are not yet well understood.[7] Hence in the last section we illustrate, with a simple normal mode analysis, features of response times which one might expect to observe.

II MATHEMATICAL MODELS

(i) The dynamic equations

We undertake a description of the liquid crystal acoustic field system on a hydrodynamic scale. For this purpose we shall utilize the hydrodynamic equations expressing conservation of mass, conservation of energy and conservation of linear momentum. One must also introduce equations to describe any additional hydrodynamic scale variables, for example, the director variables which characterize the liquid crystal. For this latter purpose one commonly introduces the "Oseen equation" in the fashion of Leslie[8] and Eriksen.[9] Alternatively one may employ the "Broken Symmetry equations" as developed by Martin *et al.*[10] and Forster,[11] or one may utilize the "torque model" of de Gennes and collaborators.[12] The advantage of the Leslie-Eriksen equations is that they purport to treat the dynamics far from an equilibrium state and their equations, correct to second order, are readily available[13] and have been utilized by several authors.[14] For rather compelling reasons, discussed by Martin *et al.*, there is a firmer theoretical basis for the "Broken Symmetry" approach. Unfortunately, the present literature contains the broken symmetry equations only in their linearized form although, in principle,[15] they could also be written out in a nonlinearized form. Certainly the elegant conceptual frame-work of the Martin and Forster approach (*e.g.*, reactive fluxes, dissipative fluxes, etc.) is well worth utilizing.[16]

In any case the equations governing the dynamics of a nematic liquid crystal are a set of seven equations [from conservation of mass (1), linear momentum (3), energy (1) and the broken symmetry equations of the director (2)]. In terms of a complete set, $\{X\}$, of hydrodynamic scale variables $\alpha_i(\mathbf{r}, t)$ they may be written formally as

$$i, = 1 \rightarrow 7; \frac{\partial}{\partial t}\alpha_i + \hat{G}(\{\alpha_i\}) + \hat{H}(\{\alpha_i \alpha_j\}) + \cdots = 0 \tag{1}$$

In $\hat{G}$ the members of the set of α_i or their spatial derivatives appear only linearily, and in $\hat{H}$ members of $\{\alpha_i\}$ or their spatial derivatives appear in a bilinear or quadratic form. We shall give further specifics of $\hat{G}$ and $\hat{H}$ as the need arises. It seems worthwhile to first discuss the method of the solution of Eq (1).

(ii) The reference state

Because of their complexity the solution of the set of equations in (1) is frequently carried out in terms of a reference state. The reference state is normally some readily obtained solution of Eq. (1) for example the equilibrium state or some other appropriate "steady state" solution. Unfortunately, solutions of Eq. (1) which would serve as appropriate reference states are sometimes not readily available! As we shall see this is certainly the case for a nematic in an acoustic field. We introduce a reference state by writing, for all i,

$$\alpha_i = \alpha_i^r + \delta\alpha_i \tag{2}$$

where the superscript r denotes the known expression for the variable α_i in the reference state. The displacement $\delta\alpha_i$ is sometimes written as a "power series,"[17] giving, for Eq. (2)

$$\alpha_i = \alpha_i^0 + \alpha_i^{(1)} + \alpha_i^{(2)} + \cdots \tag{3}$$

The precise way in which Eqs. (2) and (3) are chosen and utilized in the nematodynamic equations has a strong influence on the theoretical conclusions. Hence some simple applications would seem worthwhile at this point. For purposes of discussion imagine α_1 to be completely decoupled from the other hydrodynamic variables. We then take α_1 to be given by

$$\frac{\partial}{\partial t}\alpha_1 + a_1\alpha_1 + A\alpha_1^2 = 0 \tag{4}$$

We introduce α_1^r as a solution of this equation with some specified boundary conditions. Substitution of Eq. (2) into Eq. (4) yields

$$\frac{\partial}{\partial t}\partial\alpha_1 + (a_1 + 2A\alpha_1^r)\delta\alpha_1 + A(\delta\alpha_1)^2 = 0 \tag{5}$$

If $A\delta\alpha_1$ can be taken to be small relative to the linear coefficient then we can neglect the last term to obtain the "linearized" and homogeneous equation

$$\frac{\partial}{\partial t}\delta\alpha_1 + \Gamma_1\delta\alpha_1 = 0, \Gamma_1 = a_1 + 2A\alpha_1^r \tag{6}$$

The quantity Γ_1 displays the nonlinear heritage of this equation through $2A\alpha_1^r$. However, if the reference state is chosen such that α_1^r is zero then $2A\alpha_1^r = 0$ and the linearized equation carries no information about the nonlinearity of the original equation. If on the other hand α_1^r is non-zero this information is retained and solutions of the linearized form (Eq. (6)) can reflect the nonlinear character of the original equation. Indeed it is just this feature which allows linear stability theory to probe the solutions of non-linear equations.[18] A particularly simple illustration is the use of a steady state solution of Eq. (4) namely $\alpha_1^r = -a_1/A$. This gives $\Gamma_1 = -a_1$ so that the solution of Eq. (6) is

$$\delta\alpha_1(t) = \delta\alpha_1(0)e^{a_1 t} \tag{7}$$

The implication of Eq. (7) for $a_1 > 0$ is that $\delta\alpha_1$ grows exponentially with time so that α_1 rapidly deviates from the reference description. One concludes that this reference solution of Eq. (4) is absolutely unstable. More interesting instabilities would arise if α_1 were coupled to other variables.

Turning now to the power series solution we substitute Eq. (3) into Eq. (4) and obtain the zero, first and second order equations as

$$\frac{\partial}{\partial t}\alpha_1^{(0)} + a_1\alpha_1^{(0)} + A(\alpha^{(0)})^2 = 0 \tag{8}$$

$$\frac{\partial}{\partial t}\alpha_1^{(1)} + (a_1 + 2A\alpha_1^{(0)})\alpha_1^{(1)} = 0 \tag{9}$$

$$\frac{\partial}{\partial t}\alpha_1^{(2)} + (a_1 + 2A\alpha_1^{(0)})\alpha_1^{(2)} + A(\alpha_1^{(2)})^2 = 0 \tag{10}$$

An essential step in obtaining these equations is the assertion that $\alpha^{(m)}\alpha^{(n)}$ is of order $m + n$. Because of this assertion the *homogeneous* part of the second order equation is the same as that of the first order equation! Indeed if the reference state is such at $\alpha_1^{(0)}$ is zero then, in these homogeneous parts, all information about the nonlinearity of the original equation is lost. It is easy to see that such would be the case for the analyses of acoustic streaming which select the equilibrium state, *i.e.*, no sound field ($v^{(0)} = 0$) as the reference solution and treat the steady sound field as first order and acoustic streaming as part of the second order contribution.[17] Theoretical analyses carried out in this fashion (cf, Sripaipan *et al.*,[6] Nagai *et al.*)[7] would not even suggest an instability as arising from the homogeneous part of either the first or second order equations.

The set of nematodynamic equations are, of course, more complicated than this simple example. However, this should serve to illustrate the role of the nonlinearity and the reference state in both the linearized equation and in the first and second order equations of the power series approach.

(iii) Reference descriptions

Even if one were not interested in instabilities it is clearly of some importance to select a reference solution with some care. Certainly for reference solutions with all relevant $\alpha_i^r = 0$ the linearized analysis is sterile! Because of this various authors have utilized descriptions for which relevant α_i^r are not zero. However, at the same time, these reference descriptions may be solutions of only part of the original equation or their relation to the original equations may be less straightforward. For example, Chaban's[19] elegant analysis of the instability of a liquid crystal in an acoustic field utilizes a "pump solution"—an oscillating solution of the linear part of the original equations. On the other hand Helfrich[3] and later Nagai and Iizuka[4] introduced a reference "state" which can perhaps be best characterized as a "shear-deformed" description but its precise relation to the full nematodynamic equations is less than clear.

The implications of utilizing a reference solution $\{\alpha_i^r\}$ which is not a solution of the full equations can be illustrated by recourse once again to solutions of the simple relation given by Eq. (4). Let α_1^r be such that it is a solution of the linear part of Eq. (4) namely that

$$\frac{\partial}{\partial t}\alpha_1^r + a_1\alpha_1^r = 0 \tag{11}$$

Then substitution of $\alpha_1 = \alpha_1^r + \delta\alpha_1$ into Eq. (4) gives

$$\frac{\partial}{\partial t}\delta\alpha_1 + (a_1 + 2A\alpha_1^r)\delta\alpha_1 + A(\alpha_1^r)^2 + A(\delta\alpha_1)^2 = 0$$

The assertion that $(\delta\alpha_1)^2$ is small now leads to the *inhomogeneous* equation

$$\frac{\partial}{\partial t}\delta\alpha_1 + \Gamma_1\delta\alpha_1 + A(\alpha_1^r)^2 = 0, \Gamma_1 = a_1 + 2A\alpha_1^r \tag{12}$$

rather than the homogeneous form obtained earlier in Eq. (6). Since Eq. (11) requires that the only non-zero α_1^r are time dependent the solution of Eq. (12) becomes mathematically complicated (*cf.* Chaban[19]). In any case, the solution of the homogeneous part of Eq. (12) may indicate stability; yet, because of the inhomogeneous term, $\delta\alpha_1$ may be substantially different than zero. To see this without complicated mathematics let us treat a case where the reference state $\{\alpha_1^r\}$ is time independent and non-zero [Eq. (11) precludes this but it is easy to see that if α_1 were linearily coupled to another variable α_2 as in

$$\frac{\partial}{\partial t}\alpha_1 + a_1\alpha_1 + a_2\alpha_2 + A\alpha_1^2 + \cdots = 0$$

we could obtain a solution $\alpha_1^r \neq 0$, $\partial/\partial t\ \alpha_1^r = 0$]. Taking α_1^r to be a constant means Γ_1 is a constant and we can immediately write the solution of Eq. (12) as[23]

$$\delta\alpha_1(t) = \delta\alpha_1(0)e^{-\Gamma_1 t} - e^{-\Gamma_1 t}\int_0^t e^{\Gamma_1 t'}A(\alpha_1^r)^2 \mathrm{d}t',$$

which integrates to

$$\delta\alpha_1(t) = \delta\alpha(0)e^{-\Gamma_1 t} - \frac{A(\alpha_1^r)^2}{\Gamma_1}[1 - e^{-\Gamma_1 t}]$$

and for $t \to \infty$ and $\Gamma_1 > 0$ we have

$$\delta\alpha_1(t) \to -\frac{A}{\Gamma_1}(\alpha_1^r)^2 \quad \text{or} \quad \alpha_1 \to \alpha_1^r - \frac{A}{\Gamma}(\alpha_1^r)^2$$

Hence α_1 could, with increasing t, become quite different than just α_1^r even though Γ_1 remained positive, i.e., even though the system remains stable in terms of a decaying exponential $e^{-\Gamma_1 t}$. Of course, if $-2A\alpha_1^r$ were made sufficiently large Γ_1 could change sign, $\delta\alpha_1$, would then increase dramatically with time and, the reference α_1^r would clearly be unstable. Whether one observes only this instability or one observes the gradual growth of the contribution $A/\Gamma_1(\alpha_1^r)^2$ will depend on how α_1 is probed experimentally. The instability may occur before this latter contribution becomes observable or $A/\Gamma_1(\alpha_1^r)^2$ may be observed prior to the instability.

(iv) Boundary conditions

Although boundary conditions apply to the set of α_i, the solution is carried out in terms of the separate parts of α_i (*cf.* Eq. (2) or (3)). This presents few difficulties with α_i^r and $\delta\alpha_i$ in Eq. (2) since α_i^r usually has well defined boundary conditions. However, in expansion to second order as in Eq. (3) the selection of $\alpha_i^r = \alpha_i^{(0)}$ defines only the sum $\alpha_i^{(1)} + \alpha_i^{(2)} = \alpha_i - \alpha_i^{(0)}$. The boundary conditions on $\alpha_i^{(1)}$ and $\alpha_i^{(2)}$ can be selected arbitrarily, for example, $\alpha_i^{(1)}$ alone can be required to match the imposed acoustic field.

(v) Normal modes

Given an appropriate reference description and its boundary conditions the solution of the nematodynamic equations may be addressed. The methodology of Rayleigh,[20] followed largely, by recent workers (*cf.* Sripaipan *et al.*,[6] Nagai *et al*[7]) proceeds directly to the expression for $\alpha_i^{(m)}(t)$. We have repeatedly seen (Sections (ii) and (iii)) the role played by the solution of the homogeneous part of the dynamic equations. Where there are, in fact, several such coupled dynamic equations there are some advantages to solving the homogeneous equations in terms of their normal modes. This is particularly so in the case where the coefficients of the homogeneous equations are constants. The efficacy of a normal mode analysis is apparent even in the solution of inhomogeneous equations, for example equations of the form of Eq. (10).

We shall illustrate their use by considering the set of second order nematodynamic equations denoted by

$$\frac{\partial}{\partial t}\boldsymbol{\alpha}^{(2)} + \mathbf{M}\boldsymbol{\alpha}^{(2)} + \mathbf{F} = 0 \tag{13}$$

Here $\boldsymbol{\alpha}^{(2)}$ is a column vector with elements $\alpha_i^{(2)}$, $\mathbf{M}$ is a matrix with elements M_{ij} defined in terms of the operators $\partial/\partial x_i$ ∂/x_j and phenomenological coefficients such as those for viscosity, ν_n, heat conductivity κ, etc. The inhomogeneous part $\mathbf{F}$ is a column vector with elements F_i. This "force" F_i is defined in terms of products $\alpha_j^{(1)}\alpha_k^{(1)}$, where the $\alpha_j^{(1)}$ are solutions of the first order equation

$$\frac{\partial}{\partial t}\boldsymbol{\alpha}^{(1)} + \mathbf{M}\boldsymbol{\alpha}^{(1)} = 0 \tag{14}$$

The boundary conditions imposed on the solutions $\boldsymbol{\alpha}^{(1)}$ of Eq. (14) contain the external stresses—in this case the acoustic field. Hence the $F_i \sim \alpha_j^{(1)}\alpha_k^{(1)}$

carry information regarding the amplitude and frequency of the acoustic field. We introduce normal modes γ_j by writing[21]

$$\gamma_j(t) = \gamma_j(0)e^{-\lambda_j t} \tag{15}$$

where the $\gamma_j(0)$ are elements of a column vector

$$\boldsymbol{\gamma} = \mathbf{V}^{-1}\boldsymbol{\alpha} \tag{16}$$

Here $\mathbf{V}$ is such that

$$\mathbf{V}^{-1}\mathbf{V} = \mathbf{1} \quad \text{and} \quad \mathbf{V}^{-1}\mathbf{M}\mathbf{V} = \Lambda \tag{17}$$

and Λ is a diagonal matrix with elements $\Lambda_{jj} = \lambda_j$.

We should emphasize again that $\mathbf{M}$ is the same for both the first and second order equations. This equivalence may not be apparent from the way the nematodynamic equations are written in the literature[6] but it is easy to see from Eqs. (9) and (10). In more general terms this equivalence arises from the assertion that $\alpha_i^m \alpha_j^n$ is of order $m + n$ and the requirements we placed on $\hat{G}$ and $\hat{H}$ in Eq. (1) [both the Leslie-Eriksen equations and the generalized broken symmetry equations of Forster meet these requirements[16]].

Given the definitions of Eqs. (16) and (17) we can rewrite the set of coupled inhomogeneous equations of the second order equation as a set of *uncoupled* inhomogeneous equations. We simply multiply from the left with $\mathbf{V}^{-1}$[21] and use $\boldsymbol{\alpha} = \mathbf{V}\boldsymbol{\gamma}$ on the right of $\mathbf{M}$ to obtain

$$\frac{\partial}{\partial t}\mathbf{V}^{-1}\alpha + \mathbf{V}^{-1}\mathbf{M}\mathbf{V}\boldsymbol{\gamma} + \mathbf{V}^{-1}\mathbf{F} = 0$$

or

$$\frac{\partial}{\partial t}\gamma_j + \lambda_j\gamma_j + F_j^{\gamma} = 0 \tag{18}$$

where

$$F_j^{\gamma} = \Sigma V_{ji}^{-1} F_i \tag{19}$$

The complete solution of Eq. (18) can be immediately written down as

$$\psi_j \equiv \gamma_j(t) = \gamma_j(0)e^{-\lambda_j t} - e^{-\lambda_j t}\int_0^t e^{\lambda_j t'} F_j^{\gamma}(t')\mathrm{d}t', \tag{20}$$

(note we use ψ_j for the *complete* solution Eq. (18)). In treatments[17] where the acoustic streaming is taken to be part of the second order solution one asserts that F_j contains a constant term and the remaining terms are of less importance. Under these conditions Eq. (20) integrates to give

$$\psi_j(t) = \gamma_j(0)e^{-\gamma_j t} - \frac{F_j^\gamma}{\lambda_j}[1 - e^{-\lambda_j t}] \tag{21}$$

Consequently one can describe the solution of Eq. (13), *i.e.*, the second order equation, in terms of the "modes" ψ_j given by Eqs. (15)–(17). The description in terms of the variables $\alpha_i^{(2)}$ (*i.e.*, $v_x^{(2)}, p^{(2)}$, etc.) is obtained by simply inverting Eq. (16) to give

$$\alpha_i^{(2)}(t) = \Sigma V_{ij}\psi_j(t) \tag{22}$$

III THE CHARACTERISTICS OF THE SECOND ORDER SOLUTION

We shall discuss only those cases where the inhomogeneous term is constant and the eigenvalues λ_i of $\mathbf{M}$ have positive real parts. In these circumstances the response and long-time characteristics are of interest. From Eq. (21) we see that as $t \to \infty$ we have

$$\psi_j = -\frac{F_j^\gamma}{\lambda_j} \tag{23}$$

Thus ψ_j reaches a steady value which we might remark is simply that obtained from the second order equation (Eq. (18)) on ignoring $\partial\gamma_i/\partial t$. For the long-time steady value of say α_i we have

$$\alpha_i = \frac{\sum V_{ij}F_j^\gamma}{\lambda_j} = \frac{\sum_{jk} V_{ij}V_{jk}^{-1}F_k}{\lambda_j} \tag{24}$$

It is this solution which corresponds to the steady acoustic streaming solutions of Sripaipan *et el.*[6] We should emphasize that once one has the eigenvalues, λ_i, and eigen vectors, V_i, of the *first* order equation then all one needs to complete the solution is the "forces" F_i of the second order equation.

The response time of the second order solution can characterize either growth τ_G or decay τ_D. In the case of growth we take the initial state to have $\gamma_j(0) = 0$ [cf., Section (iv) and note that this implies all $\alpha_i^{(2)} = 0$ at time $t = 0$ so that $\alpha_i(0) = \alpha_i^{(0)}(0) + \alpha_i^{(1)}(0)$]. Hence Eq. (21) is just

$$\psi_j^G(t) = -\frac{F_j^\gamma}{\lambda_j}[1 - e^{-\lambda_j t}] \tag{25}$$

We can write τ_G as

$$\tau_G = \left|\frac{\partial\psi_j^G/\partial t}{\psi_j^G(t) - \psi_j^G(\infty)}\right|^{-1}$$

which gives for Eq. (25) $\tau_G = \lambda_j^{-1}$.

The growth of the individual $\alpha_i^{(2)}$ is of course characterized by several response times. If all $\alpha_i^{(2)}(0) = 0$ Eq. (22) gives

$$\alpha_i^{(2)} = \sum_i V_{ij}\left[\frac{F_j^{\gamma}}{\lambda_j}(1 - e^{-\gamma_j t})\right] \tag{26}$$

and in analogy with Eq. (25) we see that there are several response times $\tau_{G,j} = 1/\lambda_j$. Which of these $\tau_{G,j}$ appears dominant will depend on the elements of $(V_{ij}F_j^{\gamma}/\lambda_j)$.

Alternatively one can enquire as to the decay of, say, the steady long time value of ψ_j^{ss} when the inhomogeneous term (which we think of as a "force" which had maintained this steady state) is suddenly shut off. This means that in Eq. 21 we can take $\gamma_j(0) = \gamma_j^{ss}$ at the instant the forces F_i are set to zero and write Eq. (21) for decay (D) as

$$\psi_j^D(t) = \psi_j^{ss} e^{-\lambda_j t} \tag{27}$$

Defining the response time τ_{Dj} as

$$\left|\frac{\partial\psi_j^D/\partial t}{\psi_j^D(t) - \psi_j^D(\infty)}\right|^{-1}$$

gives $\tau_{Dj} = \lambda_j^{-1}$. As with growth, we would expect that decay of a given $\alpha_j^{(2)}$ would be characterized by several decay times as in

$$\alpha_j^{(2)} = \sum V_{ij}\gamma_j^{ss} e^{-\lambda_j t} = \sum V_{ij} V_{jk}^{-1} F_k\left(\frac{e^{-\lambda_j t}}{\lambda_j}\right) \tag{28}$$

We must remark that the decay time of a given mode ψ_j is the same as its response time. Further it should be clear that the decay times λ_j^{-1} do not depend on the "forces" F^{γ}. The λ_j are the roots of **M** and in the power series expansion utilizing an equilibrium reference (*i.e.*, no sound) the matrix **M** can be shown [*cf.* Eqs. (8) to (10) with $\alpha_1^r = 0$] to reduce to that of the equilibrium case. For this reason one would not expect this method to predict a response time λ_j^{-1} which depended on the acoustic field. However λ_j^{-1} will be sensitive to those external fields, e.g. an electric field, which appear directly in the elements of the matrix **M**.

A word of caution is in order if the response times are probed[7] in terms of the individual $\alpha_i^{(2)}$. Notice that in both Eqs. (26) and (28) the forces F_j enter as coefficients of terms with different exponential decay characteristics. Now these forces F_i do not depend in precisely the same way on a given external stress, *e.g.*, electric or even the acoustic field. Hence changing these external stresses will change the relative contributions of the various terms in these expressions for growth and decay of $\alpha_i^{(2)}$ in a complicated way. As a result the apparent decay time can depend on more than just λ_j^{-1}, it could depend on the external stress in a complicated way.[22]

The long-time behaviour and response times of acoustic streaming when the reference state is chosen as something other than equilibrium can differ from that given above. For example, if the reference state is a "pump" solution[19] or an acoustically strained description[4] then in the linearized equation,

$$\left[\frac{\partial}{\partial t}\delta\boldsymbol{\alpha} + \mathbf{M}\delta\boldsymbol{\alpha} + \mathbf{F} = 0\right],$$

where **M** now retains information about the reference state—*i.e.*, about the imposed acoustic field-in contrast to the cases discussed above. If the reference is such that **M** is time dependent the mathematics is complicated (*cf.* Chaban) but if **M** is constant it is easy to see that the analysis of the previous few paragraphs goes through in the same way. Now, however, the characteristic "response" times λ_j^{-1} could be *directly* dependent on the imposed acoustic field. Assuming stability the long time steady state results should, however, be the same.

IV CONCLUSIONS

For a nematic liquid crystal in an acoustic field the power series solution carried to second order from a reference state of zero acoustic power (*i.e.*, equilibrium) is incapable of displaying an exponential growth (instability) of the characteristic solution. Other reference descriptions could display such an instability. However, the use of reference descriptions which are *not* solutions of the full nematodynamic equations leads to solutions which may deviate significantly from the reference description yet still exhibit stability of the characteristic solution.

Depending on the mode structure the response times may show a complicated dependence on the acoustic or other external fields.

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